Chs 12, 13

12.4
Given transition from 5 to 2 for Hydrogen--find energy, frequency, wavelength.

$$E_n = -13.6 \text{ eV} * Z^2/n^2$$

$$\Delta E_{5-1} = -13.6\left(\frac{1}{5^2} - \frac{1}{2^2}\right)$$

4th excited

$$= 2.8581 \text{ eV} = h\nu$$

$$h = 4.136 \times 10^{-15} \text{ eVs}$$

$$\nu = 6.9108 \times 10^{14} \text{ Hz}$$

$$\lambda = \frac{c}{\nu} = 4.34 \times 10^{-7} \text{ m}$$

12.6 Determine the reduced mass for an electron in tritium--the nucleus has a mass of 2808.9MeV/c^2.
Then determine the wavelength for the Balmer alpha line---which is the n=3 to n=2 transition.

$$m_r = \frac{Mm}{M + m} = \frac{2808.9 * 0.511}{2808.9 + 0.511} \text{ MeV}/c^2$$

$$= 0.51091 \text{ MeV}/c^2$$

The reduced mass for normal Hydrogen is 0.51072MeV/c^2

$$\Delta E_{\substack{3-2 \\ \text{Tritium}}} = \Delta E_{\substack{3-2 \\ \text{Hydrogen} \\ \text{normal}}} * \frac{m_{r\,\text{tritium}}}{m_{\substack{\text{Hydrogen} \\ \text{normal}}}} = 1.89 \text{ eV} * \frac{0.51091}{0.51072}$$

So bigger energy, the wavelength shifts (shorter) from 656.28nm to tritium 656.04nm. And if we had some tritium (or Deuterium) to look at, we can see these shifts on a simple diffraction grating spectrometer. Recall tritium has higher energies--since the nucleus is closer to an infinite mass than normal hydrogen.

12-17

Find v as a function of c,z, n for hydrogen like atoms (one electron). Then find γ as a function of z, and expand--keeping terms only up to z^2

$$v = \frac{nh}{2\pi m r} = \frac{nh}{2\pi m r_1}\frac{z}{n^2}\frac{c}{c} \qquad r_n = \frac{n^2}{z} r_1$$

$$= \frac{z}{n}\left(\frac{h}{2\pi m r_1 c}\right) c$$

$$\simeq \frac{z}{n}\frac{1}{137} c$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx 1 + \frac{1}{2}\frac{v^2}{c^2} + \ldots$$

$$\approx 1 + \frac{1}{2}\frac{z^2}{n^2}\left(\frac{1}{137}\right)^2$$

for $n = 1$

$$\approx 1 + 2.664 \times 10^{-5} z^2$$

Things get relativistic fast.....

12-18

We previously required that a whole number of wavelengths fit in the orbit for the electron. When reduced mass is now considered, show that this condition will no longer be satisfied.

First off note that the orbital radius's are the same for the electron with either reduced mass or not. So, in order to invalidate the condition---the wavelength must change. Well, energy, thus KE, thus momentum or velocity DOES CHANGE.

Recall

$$\frac{\lambda}{2\pi r_e} = \frac{h}{2\pi r p}$$

was old condition

Whole #

$$\frac{1}{n} = \frac{h}{2\pi m_e v_e r_e} \rightarrow \underbrace{nh = 2\pi m_e v_e r_e}$$

Any MOM Quant Condition

For not reduced mass, this condition was required for a whole number of wavelengths to fit Inside the electron orbit. BUT FOR REDUCED MASS WE HAVE A DIFFERENT CONDITION ON THE QUANTIZATIOIN OF TOTAL ORBITAL ANGULAR MOMENTUM.

$$(m_e v_e r_e + \underbrace{M v_n r_n})2\pi = nh$$

$$(m_e v_e r_e + x)2\pi = nh$$

I cant have

$$\underbrace{m_e v_e r_e 2\pi} + 2\pi x = nh$$

$$\cancel{nh + 2\pi x = nh}$$

13-5 Given wavefunction, how do you find most/ least probable locations?

$$\frac{d(\psi^* \psi)}{dx} = 0 \qquad \text{save for } x$$

13.13--Given the time dep wavefunction--not normalized, how do you find probability?

$$\underbrace{P(x,t)}_{\text{prob density}} dx = \frac{\Psi^* \Psi \, dx}{\int_{\text{ALL } x} \Psi^* \Psi \, dx}$$

The denominator normalizes (or unitizes) the probability if not already given that way.

13-15 Normalize $\psi = A r e^{-r/2a_0}$ in spherical

$$1 = \int_0^\infty A^2 r^2 e^{-r/a_0} \underbrace{(4\pi r^2)}_{\text{from } \theta, \phi \text{ integrals}} dr$$

So you may use integral tables, mathematica, wolfram alpha--etc.

$$\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \qquad n = 4 \quad a = \frac{1}{a_0}$$

$$= 4!\, a_0^5$$

$$A^2 = \frac{1}{4\pi \; 4! \; a_0^5}$$

$$A = (96 \pi a_0^5)^{-1/2}$$

13-17 Given $\psi = A e^{-(x^2+y^2+z^2)/a_0}$ What is the probability of being in the +, +, + octant.

This wavefunction has the same value regardless of + or - along each axis. It is symmetric about x, y, z.
So the probability of +++ is 1/8.

13-19 Given energies---write out the time dep part of ….

$$\Psi_n = \psi_n e^{-i(E_n/\hbar)t}$$

$$= \psi_n e^{-i(n+3/2)\omega_c t}$$

13-22 If the wavefunction is uniform and non-zero show that the kinetic energy must be zero.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi/dx^2}{\psi} = \left[E - V(x)\right]$$

$$= KE$$

Left side is zero, so the KE must also be zero.
If this meant constant in time, then the energy must be zero---which turns time dependent term to "1"---and we still get this result.

Wavefunctions have time dependence and also are not spatially uniform.